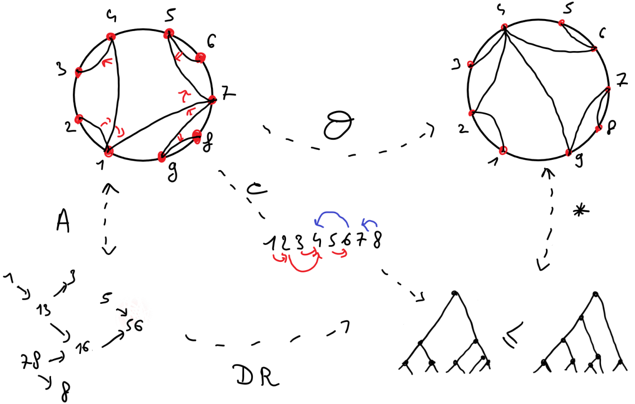
Hidden symmetry of the Tamari lattices

Baptiste Rognerud



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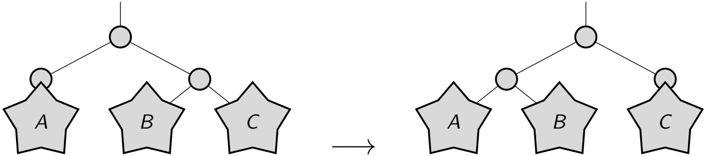

FIGURE – Right rotation.

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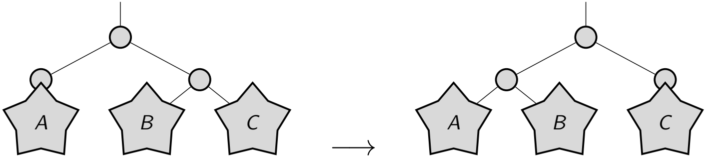

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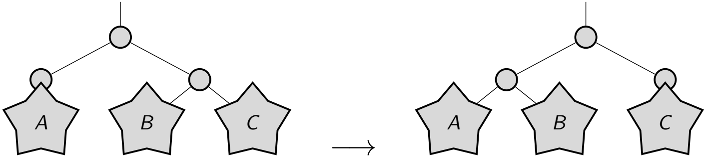

FIGURE – Right rotation.

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Theorem (Chapoton 2007,R-2020)

The Coxeter matrix of Tam_n *satisfies* $C^{2n+2} = Id$.

Coxeter transformation

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- During the rest of the talk I will work with $-\Theta$!

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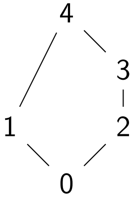

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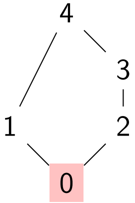

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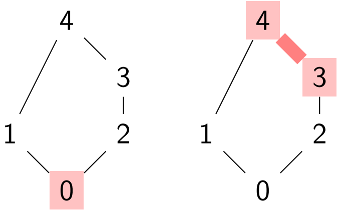

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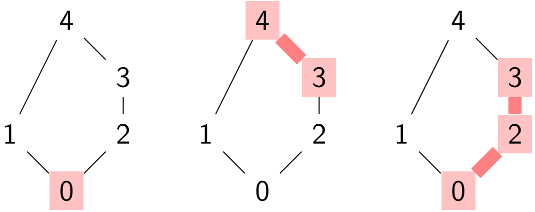

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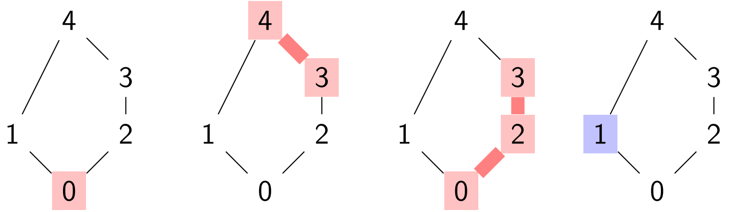

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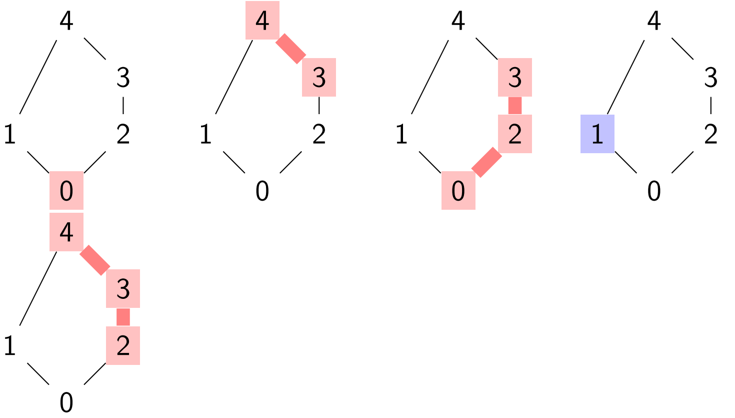

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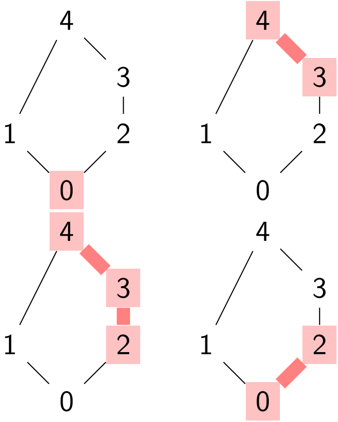

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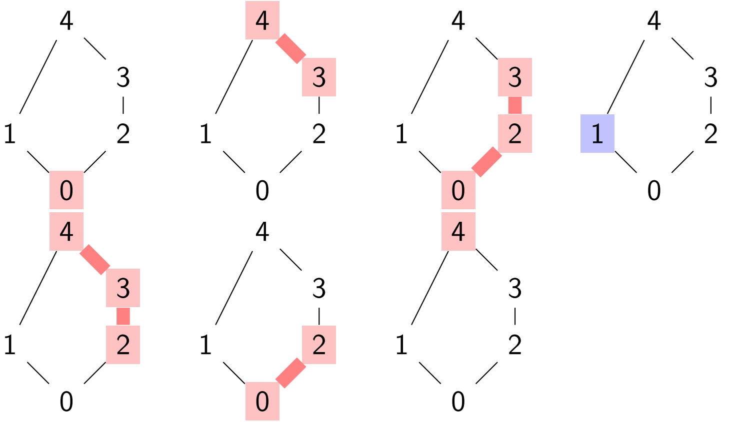

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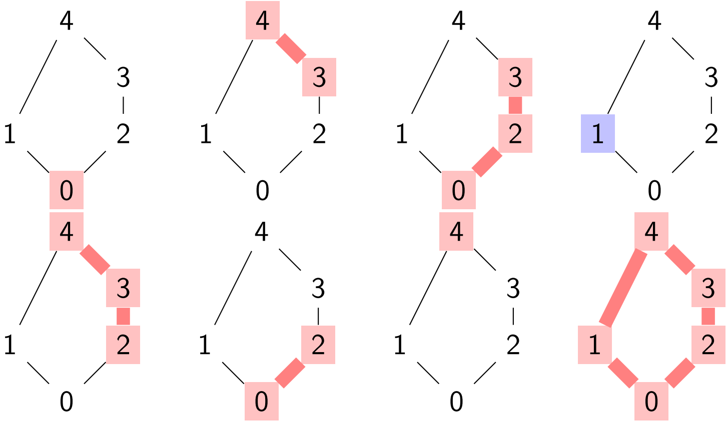

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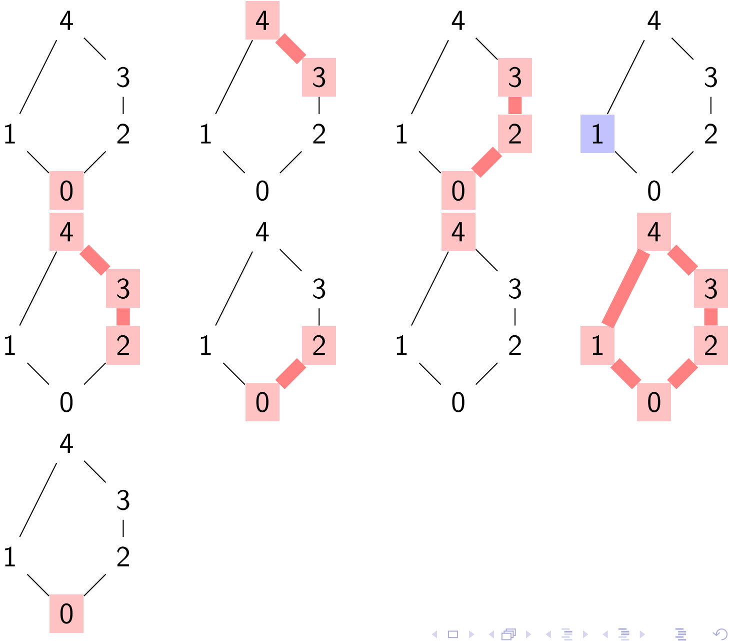

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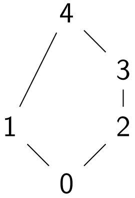

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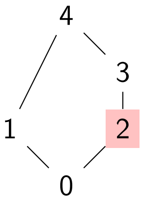

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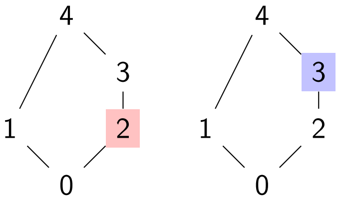

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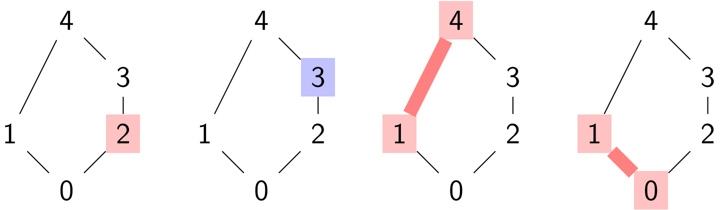

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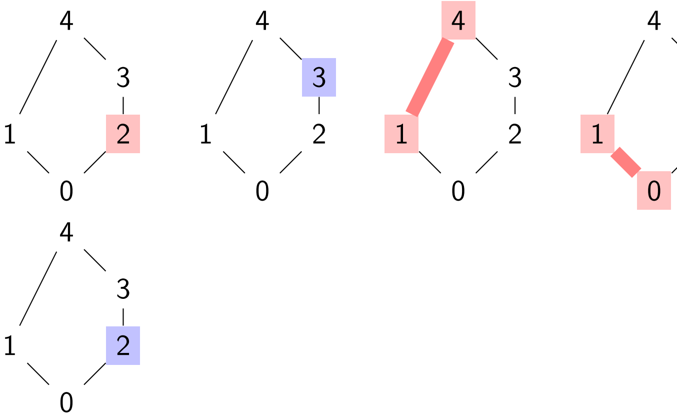

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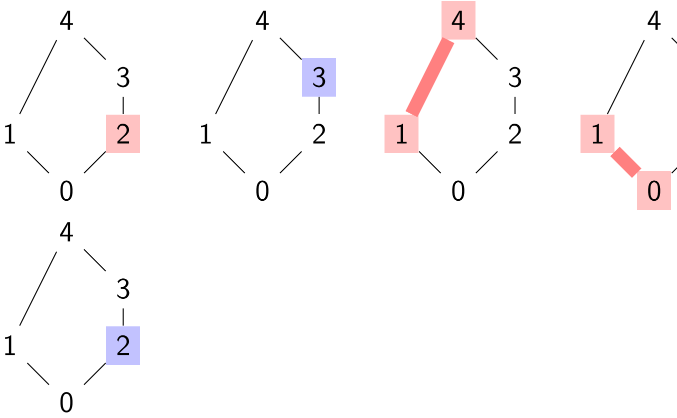

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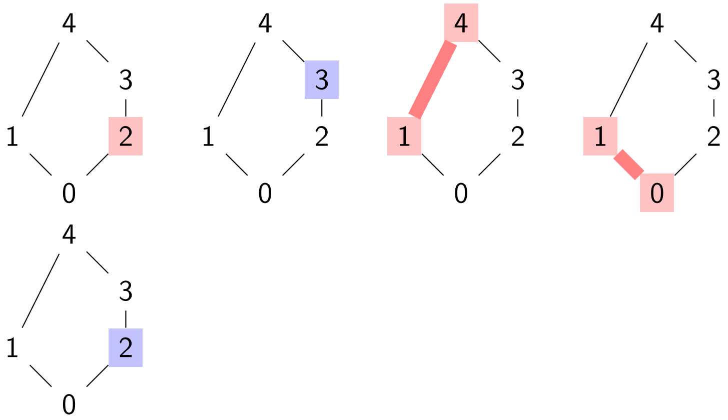

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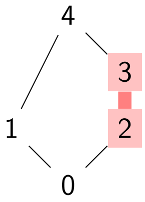

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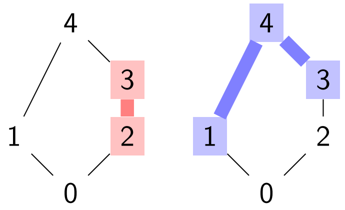

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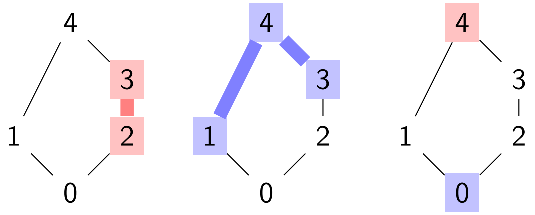

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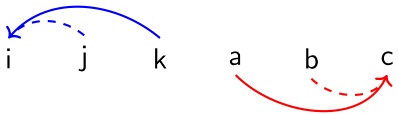

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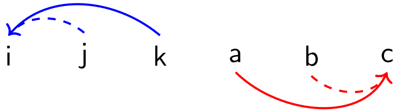

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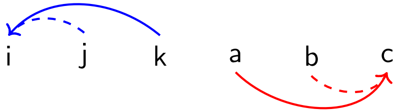

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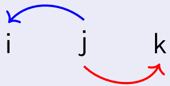

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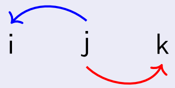

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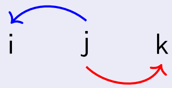

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Theorem (Chapoton, CNHT,R)

There are two injective maps, a bijection, a commutative triangle $\mathrm{NCT}_n \overset{C}{\hookrightarrow} \mathrm{Int}(\mathrm{Tam}_n)$, $\mathrm{NCT}_n \xrightarrow[1:1]{\Psi} \text{Exc-IP}$, $\text{Exc-IP} \overset{CP}{\hookrightarrow} \mathrm{Int}(\mathrm{Tam}_n)$

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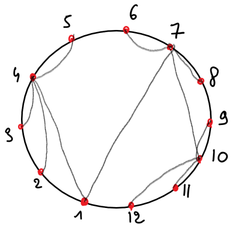

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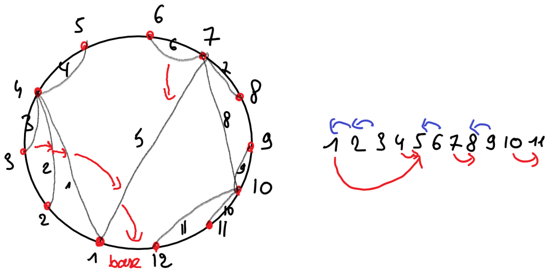

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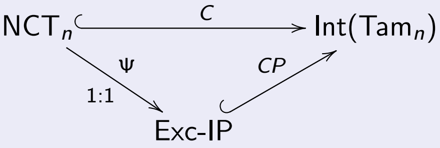

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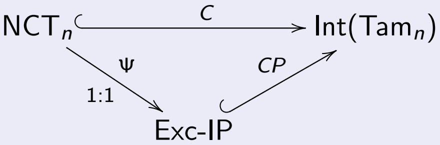

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Via brackets vectors

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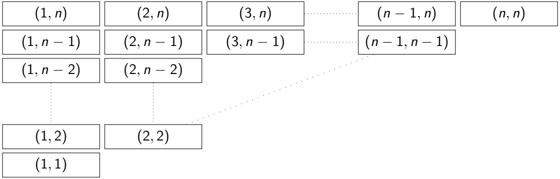

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$(1,n-2)$	$(2,n-2)$				
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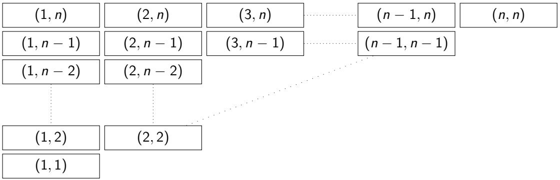

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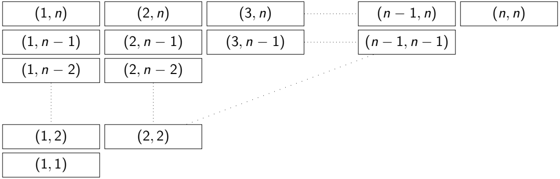

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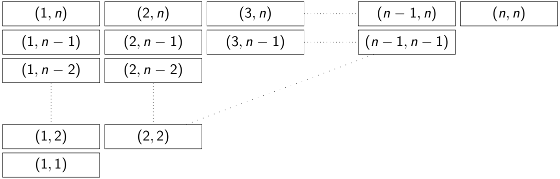

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- Example of A_2.
- $(\mathrm{Tors}(A_n), \subseteq) \cong \mathrm{Tam}_{n+1}$ (via brackets vectors).

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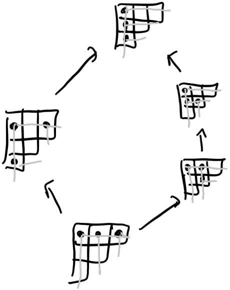

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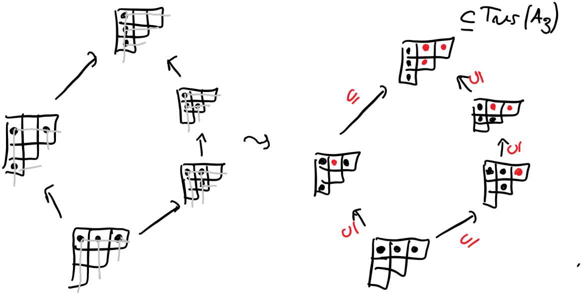

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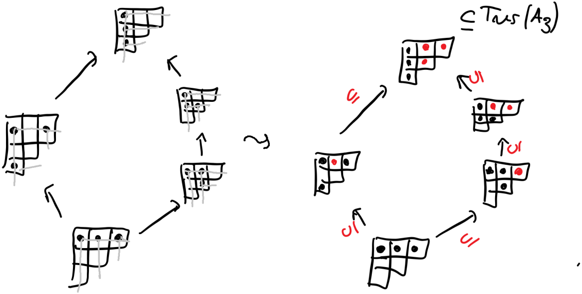
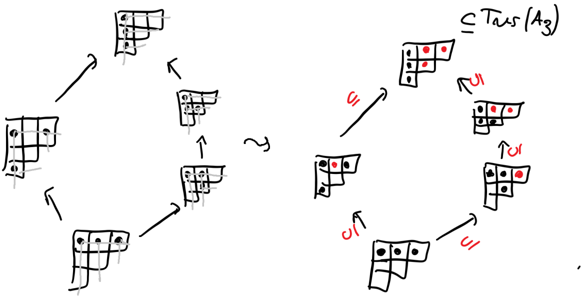


- Poset $T_1 \leq T_2$ if $\mathrm{Fac}(T_1) \subseteq \mathrm{Fac}(T_2)$.

Via binary trees

- $\text{Tilt}(A_n) \cong \text{Tam}_n$ (via binary trees).
- $\text{Tilt}(A_n) \cong \text{Tam}_n$ is an interval in $\text{Tors}(A_n) \cong \text{Tam}_{n+1}$.
- Example of A_3.



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There is a commutative square of injections and bijections

$$\begin{array}{ccc} \mathrm{NCT}_n & \xhookrightarrow{C} & \mathrm{Int}(\mathrm{Tam}_n) \\ {\scriptstyle Araya}\downarrow{\scriptstyle 1:1} & & \downarrow{\scriptstyle 1:1} \\ \mathrm{Exc}(A_n) & \xhookrightarrow{DR} & \mathrm{Int}(\mathrm{Tilt}(A_n)) \end{array}$$

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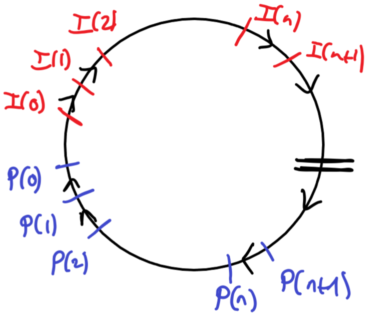
DR sends an exceptional collection $\mathcal{E}$ to $\mathbb{I}_{\mathcal{E}} = [I_{\mathcal{E}}, P_{\mathcal{E}}]$ where $I_{\mathcal{E}}$ and $P_{\mathcal{E}}$ are the minimal injective cogenerator and projective generator of the category $\mathcal{F}(\mathcal{E})$.

An example

$A := k(1 \rightrightarrows 2)$

"$\mathbb{N} < \mathbb{N}^{op}$"

$\mathrm{Tilt}(A) =$



$\underline{Exc} = -[x, x] \ \forall x \in \mathrm{Tilt}(A)$

$- [I(0), P(0)]$

Exceptional intervals

Theorem (R 2018,2023)

For an interval I of Tam_n, TFAE

- *I is in the image of C.*
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Theorem (DIRT, BCZ,A,... ?)

- *The Tamari lattice* Tam_n *is* *semidistributive* ($x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ *whenever* $x \wedge y = x \wedge z$ *and dual).*

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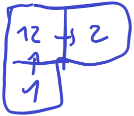
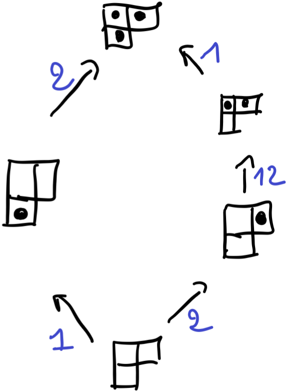
- *The join-irreducibles are in bijection with the intervals of* $[n]$ *(the indecomposables modules of* A_n*.)*
- *For torsion classes,* $A \subseteq B$ *is labelled by the unique interval* I *in* B *such that* $\mathrm{Hom}(X, I) = 0 \;\; \forall X \in A$*.*

Example of Tam_3





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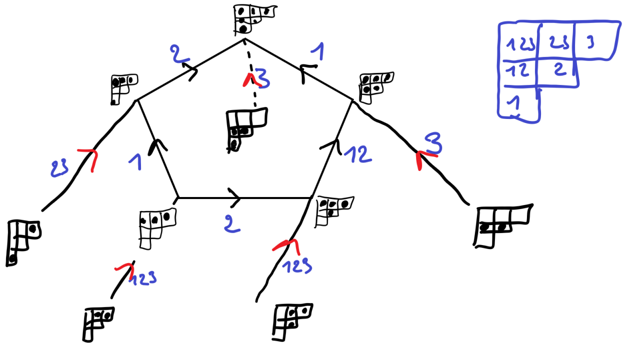

Walls of an intervals

Definition

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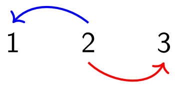

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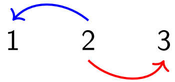

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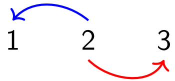

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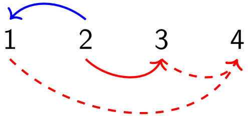

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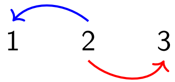

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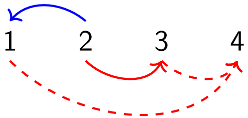

Theorem (R 2023)

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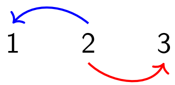

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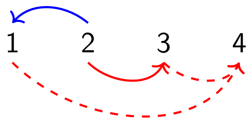

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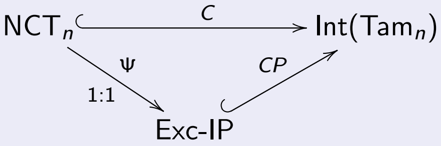
- *The number of walls is n + the number of ordinary* configurations.*

Hidden symmetry (ii)

Theorem (Chapoton, CNHT,R)

There are two injective maps, a bijection, a commutative triangle



Theorem (R 2020/2023)

The Coxeter transformation of Tam_n *sends an exceptional interval* I *to* $C(Walls(I))$.

n \ extra walls	0	1	2	3	4	5
2	3					
3	12	1				
4	55	12	1			
5	273	105	19	2		
6	1428	816	234	48	4	
7	7752	5985	2380	716	123	9

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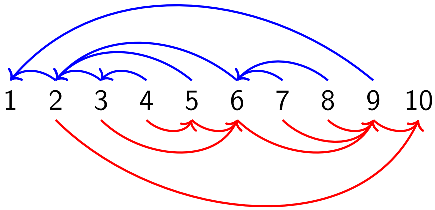

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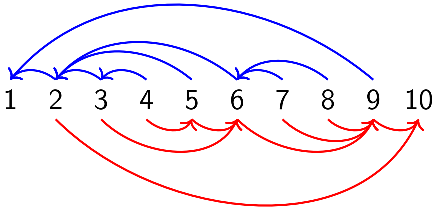

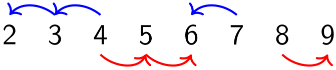

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